$$EU = \frac{\hat{\mu}\left(\hat{\mu}^2 n_1 n_2 + \hat{\lambda}^2 (1+n_1) n_2 + \hat{\lambda}\hat{\mu} n_1 (-1+2n_2)\right)}{(\hat{\lambda}+\hat{\mu})^3 n_1 n_2};$$

$$EU2 = \left(\hat{\mu}^2 \left(2\hat{\lambda}\hat{\mu} n_1 (-1+n_2) + \hat{\mu}^2 n_1 n_2 + \hat{\lambda}^2 \left(3n_2 + n_1 (1+n_2)\right)\right)\right) \Big/ \left((\hat{\lambda}+\hat{\mu})^4 n_1 n_2\right);$$

```
VarU = Simplify[EU2 - EU^2];
```

```
z = Quantile[NormalDistribution[0, 1], 1 - α / 2];
w = 2 * z * Sqrt[VarU];
```

```
λ = 1;
μ = 100;
tu = N[μ / (λ + μ)]
α = 0.01;
β = 0.01;
```

```
0.990099
```

```
ns = 10;
λ̂ = ns / Total[RandomVariate[ExponentialDistribution[λ], ns]]
μ̂ = ns / Total[RandomVariate[ExponentialDistribution[μ], ns]]
```

```
1.82203
```

```
162.861
```

```
n1 =.;
n2 =.;
L = n1 + n2 - k * (w / EU - β);
Solve[{D[L, n1] == 0, D[L, n2] == 0, w / EU - β == 0}, {n1, n2, k}, Reals]
```

$$\{\{n_1 \to -5.12845,\ n_2 \to 2.93977,\ k \to -159.513\},$$
$$\{n_1 \to -2.11307 \times 10^{-20},\ n_2 \to -1.88875 \times 10^{-18},\ k \to 0. - 1.34605 \times 10^{-44}\, i\},$$
$$\{n_1 \to 0.00049517,\ n_2 \to -0.0442625,\ k \to -0.000301347\},$$
$$\{n_1 \to 64.9992,\ n_2 \to 64.0213,\ k \to -25\,992.5\}\}$$

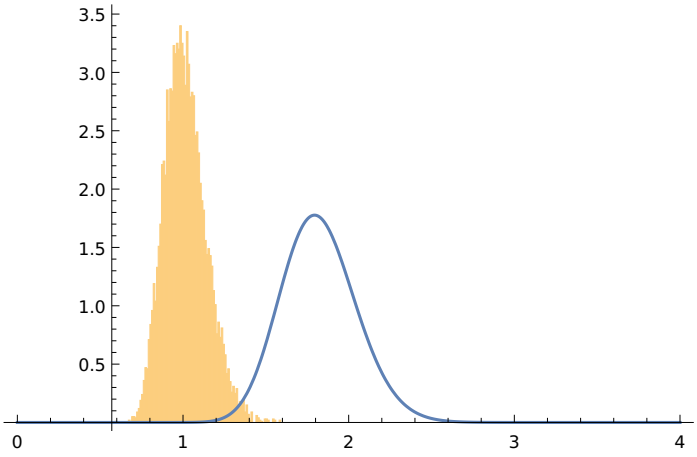
Verify that n_1 and n_2 is the correct solution.

```
w / EU /. {n1 → 65, n2 → 64}
```

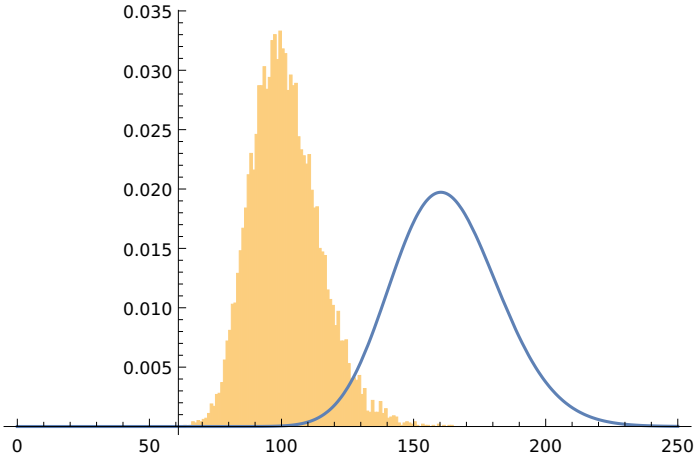
```
0.0100008
```

Verify the correctness of this method by sampling:
1. The poster distribution
2. Distribution of U

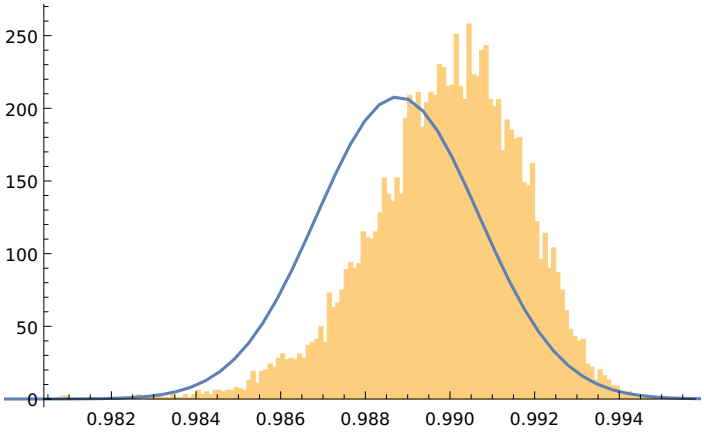
```
n1 = 65;
n2 = 64;
Λ = GammaDistribution[n1, λ̂ / n1];
M = GammaDistribution[n2, μ̂ / n2];
U = TransformedDistribution[μ/(λ + μ), {λ, μ} ≈ ProductDistribution[Λ, M]];
lmdSamples =
  Table[n1 / Total[RandomVariate[ExponentialDistribution[λ], n1]], 10000];
Show[Histogram[lmdSamples, 100, "PDF"],
 Plot[PDF[Λ, t], {t, 0, 4}], PlotRange → All]
```




```
muSamples =
  Table[n2 / Total[RandomVariate[ExponentialDistribution[μ], n2]], 10000];
Show[Histogram[muSamples, 100, "PDF"],
 Plot[PDF[M, t], {t, 0, 250}], PlotRange → All]
```



```
uSamples =
  Table[Module[{l = n1 / Total[RandomVariate[ExponentialDistribution[λ], n1]],
     m = n2 / Total[RandomVariate[ExponentialDistribution[μ], n2] ]},
    m/(l + m)], 10 000];
Show[Histogram[uSamples, 100, "PDF"],
 Plot[PDF[NormalDistribution[EU, Sqrt[VarU]], t], {t, 0, 1}]]
tu
empU = EmpiricalDistribution[uSamples];
{tu * (1 + β/2), tu * (1 - β/2)}
1 - (CDF[empU, tu * (1 + β/2)] - CDF[empU, tu * (1 - β/2)])
α
```




```
0.990099

{0.99505, 0.985149}

0.0098

0.01
```